Topology Optimization of Rotor Structure for Synchronous Motor Using the Method of Moving Asymptotes

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Topology optimization has superior priority from the point of view of degree of structural freedom in comparison to the shape optimization and hence there are good possibilities to derive new structure of electrical machinery. To identify the reasonable topology in which the gray scale exists slightly, the Heaviside function is suitable for the characteristic functions. Furthermore, when the method of moving asymptotes (MMA) is applied to topology optimization method, a drastic improvement in convergence characteristics is achieved. Therefore, the potential of topology optimization application for the practical design of electrical machinery can be graduallyincreased.

In this paper, advanced performance of the synchronous reluctance motor (SynRM) is targeted using topology optimization method. While the manufacturing cost for the SynRM is considerably lower than that of the permanent magnet type synchronous motor, its torque characteristics are poor. In order to overcome this difficulty, the topology optimization implemented by using Heaviside function and MMA is applied to SynRM.

***Index Terms*—Finite element analysis, method of moving asymptotes, synchronous reluctance motor, and topology optimization.**

I. Introduction

Topology optimization (TO) method is more advantageous when compared with the shape optimization method, from the point of view of high degree of freedom in shape, which is due to the free boundary and independence from the conventional structure in electrical machinery. In the MMA-based TO method [1], Heaviside function is adopted as the characteristic function and the convergence acceleration supported by the method of moving asymptotes (MMA) [2] that is also incorporated. The performance of this TO method is demonstrated in the 3-D magnetic shielding problem as compared with the conventional level set method [3], [4]. However, the affinity of its method for the synchronous motors is not clearly verified.

The demand of synchronous reluctance motor (SynRM) are actively increasing due to its low manufacturing cost. Hence, to enhance the practicability of the MMA-based TO method in the synchronous motor. In this paper, some effective structures of rotor core of SynRM are demonstrated using MMA-based TO method.

II. Methodology of Topology Optimization

A. Nonlinear Reluctivity Using Heaviside Function

For TO, nonlinear magnetic reluctivity $\nu(\psi)$ in the design domain is formulated as follows:

$$\nu(\psi) = \{1 - H(\psi)\}\nu_0 + H(\psi)\nu_e(\boldsymbol{B}^2) \ , \tag{1}$$

where ψ is the design variable, $\boldsymbol{B}$ is the magnetic flux density, ν_0 is the reluctivity of vacuum, $\nu_e(\boldsymbol{B}^2)$ is the nonlinear reluctivity of the iron core which is evaluated from the nonlinear B-H curve, and $H(\psi)$ is the smoothened Heaviside function which determines the distribution of magnetic body using the parameter ψ in the design domain. Then, $H(\psi)$ can be shown as follows:

$$H(\psi) = \begin{cases} 0 & (\psi < -h) \\ \dfrac{3}{16}\left(\dfrac{\psi}{h}\right)^5 - \dfrac{5}{8}\left(\dfrac{\psi}{h}\right)^3 + \dfrac{15}{16}\dfrac{\psi}{h} + \dfrac{1}{2} & (-h \le \psi \le h) \\ 1 & (h < \psi) \end{cases} , \tag{2}$$

where h is the transition width that continuously connects the interval between $H(\psi) = 0$ and $H(\psi) = 1$.

B. Subproblem Derived from MMA

The optimization problem, which is composed of a minimized function $f_0(\psi)$ and few constraint functions $f_i(\psi)$, is formulated as:

$$\begin{aligned} &\min. \quad f_0(\psi) \\ &\text{s. t.} \quad f_i(\psi) \le f_{i0}, \quad (i = 1, 2, \cdots, m) \ , \\ &\qquad\quad -h \le \psi_j \le h, \quad (j = 1, 2, \cdots, n) \end{aligned} \tag{3}$$

where m is the number of constraint condition, and n is the number of ψ. Further, all functions $f_i(\psi)$ at k^{th} optimization step ($\psi = \psi^{(k)}$) are expanded using y_i, which is the inverse of the linear function as follows:

$$F_i^{(k)}(\psi) = f_i(\psi^{(k)}) + \sum_{j=1}^{n} \left.\frac{\partial f_i}{\partial y_j}\right|_{\psi^{(k)}} \{y_j(\psi) - y_j(\psi^{(k)})\} \ , \tag{4}$$

where the function y_i is defined as:

$$y_j = \begin{cases} \dfrac{1}{u_j^{(k)} - \psi_j} & \left(\partial f_i / \partial \psi_j\big|_{\psi_j^{(k)}} > 0\right) \\ \dfrac{1}{\psi_j - l_j^{(k)}} & \left(\partial f_i / \partial \psi_j\big|_{\psi_j^{(k)}} < 0\right) \end{cases} , \tag{5}$$

where $u_j^{(k)}$ and $l_j^{(k)}$ are the upper and lower asymptotes, respectively, which are adaptively determined [2]. Further, the subproblem at the k^{th} iteration is solved by using the dual method. Since the expanded functions are convex functions, the convergence speed of TO method is relatively faster than that of the conventional level set method [3], [4].

III. Analysis Model

Fig. 1 shows the analysis model of a SynRM [5]. The optimization target of this model is to improve the torque characteristics under the condition that the area of magnetic body $S_i(\Psi)$ in design domain Ω_d is less than the constraint value S_0. The optimization problem is formulated as follows:

$$\begin{aligned} &\text{min.} \quad W_1(\psi) \text{ or } W_2(\psi) \\ &\text{s. t.} \quad S_i(\psi) = \int_{\Omega_d} H(\psi)\, dS \le S_0 \\ &\qquad -h \le \psi_j \le h, \quad (j = 1, 2, \cdots, n) \end{aligned} \tag{6}$$

Here, the first minimized function $W_1(\Psi)$ is applied to improve the average torque T_a value. On the other hand, the function $W_2(\Psi)$ is applied for uniformization of the torque characteristics. Both $W_1(\Psi)$ and $W_2(\Psi)$ are formulated as:

$$W_1(\psi) = -T_a\,, \tag{7}$$

$$W_2(\psi) = \sum_i (T_i - T_0)^2\,, \tag{8}$$

where T_a is the average torque, T_i is the torque evaluated at the i^{th} rotor position, and T_0 is the target torque. The rotating pitch of rotor is set to 1°, and $W_1(\Psi)$ and $W_2(\Psi)$ are evaluated by 2-D nonlinear magnetostatic field analyses through 30°. The mirror symmetry condition on the design domain Ω_d has been indicated in green color as shown in Fig. 1. Due to this condition, the actual design domain is half of Ω_d. The number of finite element is 11,656, and DoF is 11,729. To maximize the torque characteristics in advance, the current phase angle of magnetizing winding is fixed to 45° during the optimization process.

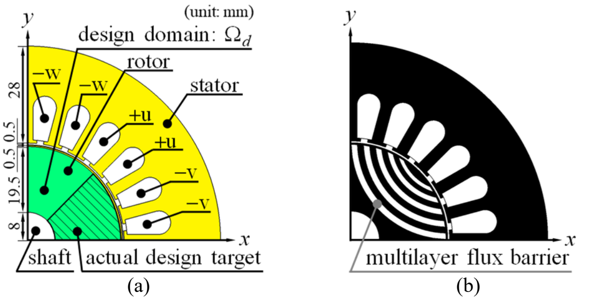

Fig. 1. Analysis model of SynRM: (a) optimization target, and (b) reference.

IV. Optimization Results

TABLE I lists the optimization parameters. The parameter ε_{opt} is the value used for convergence criterion as:

$$\left| \frac{W(\psi^{(k+1)}) - W(\psi^{(k)})}{W(\psi^{(0)})} \right| < \varepsilon_{\text{opt}}\,, \tag{9}$$

where the superscript describes the iteration number of optimization. The initial structure of optimization is set to gray scale as shown in TABLE I ($\psi^{(0)} = 0$).

Fig. 2 shows the optimized structure of rotor core. When TO to minimize $W_1(\psi)$ is carried out, the central region of the rotor is perforated as shown in Fig. 2 (a). Similarly, perforation is achieved in minimization of $W_2(\psi)$. Occurrence of some protrusions can be noticed on both tips implying improvement in the lower torque. Fig. 3 shows the torque characteristics. While the torque characteristics of min. $W_1(\psi)$ oscillate more compared to that of the reference model, the characteristics derived from min. $W_2(\psi)$ are uniformized.

TABLE I Optimization Parameters

min. func.	n	h [mm]	S_0 [m²]	$\Psi^{(0)}$	T_0 [Nm]	ε_{opt}
$W_1(\psi)$	2,876	2.0	2.5×10^{-4}	0	–	2.0×10^{-3}
$W_2(\psi)$					12.0	1.0×10^{-6}

TABLE II lists the optimization results. The parameter k_{opt} is the elapsed iteration of TO. In both the cases of minimized $W_1(\psi)$ and $W_2(\psi)$, volume of the rotor could be saved owing to strong constraint conditions. The uniformity in minimization of $W_2(\psi)$ is better than that of the reference. Further practical conditions will be applied for the upcoming stages of fullpaper.

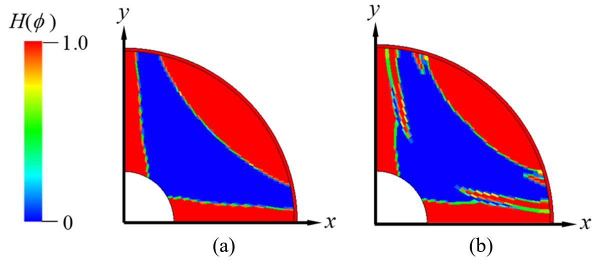

Fig. 2. Optimized rotor structure: (a) min. $W_1(\psi)$, and (b) min. $W_2(\psi)$.

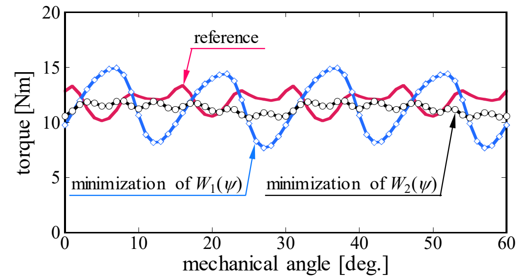

Fig. 3. Torque characteristics.

TABLE II Optimization Results

opt. model	k_{opt}	T_a [Nm]	$\sum_i (T_i - T_0)^2$	S_i [m²]	elapsed time [h]
reference	–	11.96	23.7	3.0×10^{-4}	–
min. $W_1(\psi)$	189	11.53	180.9	2.5×10^{-4}	7.8
min. $W_2(\psi)$	188	11.30	20.9	2.5×10^{-4}	8.4

CPU: Intel Core i7-6850K 3.6 GHz & 128 GB

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